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SPECTRAL ANALOGY BETWEEN TEMPERATURE AND VELOCITY FLUCTUATIONS IN SEVERAL TURBULENT FLOWS

L. FULACHIER
Institut de Mécanique Statistique de la Turbulence, LA CNRS N° 130, Université d'Aix-Marseille, 13003 Marseille, France

and

R. A. ANTONIA
Department of Mechanical Engineering, University of Newcastle, N.S.W. 2308, Australia

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Abstract—The analogy between the spectral distributions of the temperature variance and the turbulent kinetic energy, established by Fulachier and Dumas in the case of a turbulent boundary layer with zero pressure gradient over a uniformly slightly heated surface, is tested in different turbulent flows. These flows include a boundary layer subjected to the sudden application of wall suction, a moderately unstable boundary layer over a rough wall, the atmospheric surface layer at very high Reynolds numbers and with different stability conditions, a plane jet, a plane wake and nearly homogeneous turbulence with uniform mean velocity and mean temperature gradients. The analogy works well except in the atmospheric surface layer when conditions become increasingly stable. In the case of natural convection and supersonic boundary layer flows, the information on the velocity field is incomplete but, the available results do not invalidate the analogy.

NOMENCLATURE

d	nozzle width or cylinder diameter
f	normalized frequency, $ny/\bar{U}$
F_β	spectral density of β ($\equiv u, v, w$ or θ)
F_q	spectral distributions of $\overline{q^2}$, defined by equation (1)
Gr	Grashof number, $g\eta(T_w - T_1)x^3/\nu^2$
g	gravitational acceleration
h	height or width of the shear flow generator
k_1	one-dimensional wave number, $2\pi n/\bar{U}$
l	similarity length scale for plane wake, $(x/d)^{1/2}$
L	Monin–Obukhov length, $U_*^2 T/\kappa g T_*$
L_u	mean velocity half-width
M	Mach number
n	frequency
p	pressure fluctuation
Pr	molecular Prandtl number, ν/α
$\overline{q^2}$	twice the turbulent kinetic energy per unit mass ($\equiv \overline{u^2} + \overline{v^2} + \overline{w^2}$)
$\tilde{q}$ or u_i	fluctuation velocity vector with components u_1 ($\equiv u$), u_2 ($\equiv v$), u_3 ($\equiv w$)
Re_d	Reynolds number, $U_1 d/\nu$ (wake) or $U_j d/\nu$ (jet)
Re_δ	Reynolds number, $U_1\delta/\nu$
Re_λ	turbulent Reynolds number, $\overline{u^2}^{1/2}\lambda_u/\nu$
$\bar{T}$	local mean temperature
u, v, w	velocity fluctuations in x-, y-, z-directions, respectively
$\overline{u_i(t)u_j(t+\tau)}$	correlation between u_i and u_j
$\bar{U}$	local mean velocity in x-direction
U_j	jet exit velocity
U_0	mean velocity on jet centreline
V_w	suction velocity
x, y, z	coordinates: x, streamwise; y, normal to the surface (even in atmosphere) or the plane of symmetry; z, spanwise
y_*	normalized distance from the surface, yU_*/ν.

Greek symbols

α	thermal diffusivity
λ	ratio of specific heats
δ	edge of momentum boundary layer in ref. [7], $\bar{U} \simeq U_1$
δ_{th}	edge of thermal boundary layer in ref. [7], $\bar{T} \simeq T_1$
$\varepsilon, \varepsilon_\theta$	dissipations of $\overline{q^2}/2$ and $\overline{\theta^2}/2$, respectively
η	coefficient of thermal expansion
θ	temperature fluctuation
θ_0	stagnation temperature fluctuation
κ	von Kármán constant
λ_u	longitudinal microscale
ν	kinematic viscosity
ρ	density of air
τ	time delay in correlation function
ω	circular frequency, $2\pi n$
ω^*	normalized circular frequency, $\omega L_u/U_0$ (plane jet) or $\omega l/U_1$ (wake).

Subscripts

1	free stream
w	wall
*	friction.

1. INTRODUCTION

IT HAS been noted that in turbulent flows the instantaneous temperature is convected by the instantaneous velocity vector, with molecular diffusion smoothing out only small-scale temperature fluctuations [1,2]. The transport equation for the temperature fluctuation θ underlines the fact that θ depends on the velocity vector. It seems, therefore, reasonable to expect the statistics of θ to depend on those of the velocity vector rather than on any particular velocity fluctuation component. Lumley and Panofsky [2] noted that temperature spectra exhibited maxima which "are intermediate between those of u and v, suggesting that both horizontal and vertical velocity fluctuations contribute to the fluctuations of temperature". On the basis of measured spectra and co-spectra in a slightly heated turbulent boundary layer, Fulachier [3] noted that θ depends primarily on the longitudinal velocity fluctuation u at small frequencies and on the normal velocity fluctuation, v at higher frequencies. These observations led to an obvious question: was there a velocity spectrum which was analogous to the temperature spectrum? More specifically, was there a quantity characteristic, in some measure, of the velocity field which was analogous to the auto-correlation $\overline{\theta(t)\theta(t+\tau)}$? If the velocity correlation tensor $\overline{u_i(t)u_j(t+\tau)}$ is considered, the first invariant† $\overline{u_i(t)u_i(t+\tau)}$ which can be re-written as $\overline{\tilde{q}(t)\cdot\tilde{q}(t+\tau)}$ where $\tilde{q}$ is the velocity fluctuation vector, has a simple physical meaning: it is the turbulent kinetic energy.

Transport equations‡ for the correlations $\overline{\theta(t)\theta(t+\tau)}$ and $\overline{\tilde{q}(t)\cdot\tilde{q}(t+\tau)}$ can be written for a constant density flow (using tensor notation) as

$$\bar{U}_i\frac{\partial}{\partial x_i}\overline{\theta(t)\theta(t+\tau)}+\{\overline{u_i(t)\theta(t+\tau)}+\overline{u_i(t+\tau)\theta(t)}\}\frac{\partial\bar{T}}{\partial x_i}$$
$$+\overline{\theta(t+\tau)\frac{\partial}{\partial x_i}\{u_i(t)\theta(t)\}}+\overline{\theta(t)\frac{\partial}{\partial x_i}\{u_i(t+\tau)\theta(t+\tau)\}}$$
$$-\alpha\left(\overline{\theta(t+\tau)\frac{\partial^2\theta(t)}{\partial x_i^2}}+\overline{\theta(t)\frac{\partial^2\theta(t+\tau)}{\partial x_i^2}}\right)=0. \quad (1)$$

The corresponding equations for $\tilde{q}$ or u_i (note $u_i^2\equiv q^2$) are

$$\bar{U}_i\frac{\partial}{\partial x_i}\overline{u_j(t)u_j(t+\tau)}+\{\overline{u_i(t)u_j(t+\tau)}+\overline{u_i(t+\tau)u_j(t)}\}\frac{\partial\bar{U}_j}{\partial x_i}$$
$$+\overline{u_j(t+\tau)\frac{\partial}{\partial x_i}\{u_i(t)u_j(t)\}}+\overline{u_j(t)\frac{\partial}{\partial x_i}\{u_i(t+\tau)u_j(t+\tau)\}}$$
$$-\nu\left(\overline{u_j(t+\tau)\left(\frac{\partial^2u_j(t)}{\partial x_i^2}+\frac{\partial^2u_i(t)}{\partial x_i\partial x_j}\right)}+\overline{u_j(t)\left(\frac{\partial^2u_j(t+\tau)}{\partial x_i^2}+\frac{\partial^2u_i(t+\tau)}{\partial x_i\partial x_j}\right)}\right)$$
$$+\left(\overline{u_j(t+\tau)\frac{\partial p(t)}{\partial x_j}}+\overline{u_j(t)\frac{\partial p(t+\tau)}{\partial x_j}}\right)=0. \quad (2)$$

For a molecular Prandtl number Pr equal to unity, equations (1) and (2) are formally analogous except for the last term in equation (2), which contains the kinematic pressure fluctuation p. For homogeneous turbulence, this term disappears as $\overline{\partial(u_jp)/\partial x_j}\equiv\overline{u_j\,\partial p/\partial x_j}$ because of continuity. So, for this special case, and $Pr=1$, the equations are formally analogous. It follows that equations for the temperature spectrum F_θ, the Fourier transform of $\overline{\theta(t)\theta(t+\tau)}$, and for the spectrum F_q, the Fourier transform of $\overline{u_j(t)u_j(t+\tau)}$, are also analogous for homogeneous turbulence. The spectrum $F_q(n)$§ can be written in terms of the spectra of the individual velocity fluctuations u_1 ($\equiv u$), u_2 ($\equiv v$), and u_3 ($\equiv w$)

$$\overline{q^2}F_q(n)=\overline{u^2}F_u(n)+\overline{v^2}F_v(n)+\overline{w^2}F_w(n). \quad (3)$$

Fulachier and Dumas [5,6] and Fulachier [7] found that the analogy between F_θ and F_q was valid across almost the whole boundary layer and applied closely to the part of the spectrum which accounted for at least 80% of $\overline{\theta^2}$ or $\overline{q^2}$. This result is of particular interest since, although there are analyses which predict the shape of the temperature spectrum in the inertial subrange and at higher frequencies [1,8,9], there is no such analysis at lower frequencies where the temperature field is very energetic and strongly anisotropic. This frequency range may also be sensitive to initial and boundary conditions; consequently, a variation in spectral behaviour may be expected in different flows or even the same flow as a result of a change in these conditions. It should, however, be noted that, for isotropic turbulence, the feasibility of large-eddy simulation has been demonstrated for the study of the decay of passive temperature fluctuations [10].

The spectral analogy is supported mathematically via equations (1) and (2). From a physical point of view, an analogy between $\overline{\theta^2}$ and $\overline{q^2}$ seems as plausible as the Reynolds analogy. If molecular effects are assumed not to affect the energy containing part of the spectrum, it may be postulated that temporal changes in θ reflect those of another scalar quantity which represents the fluctuating velocity field. The quantity $\overline{q^2}$ may be interpreted as representing the average strength of the

† Note that it is the spectrum of the total energy which is referred to as the three-dimensional (3-D) spectrum as it is the sum of the diagonal components of the Fourier transform of $\overline{u_iu_j}$ which is of primary interest [4].

‡ Equation (1) is simply obtained by multiplying the equation for the temperature fluctuation at time t by $\theta(t+\tau)$ and adding it to the equation for the temperature fluctuation at time $t+\tau$ multiplied by $\theta(t)$.

§ The spectrum of $\tilde{q}$, for example, is a tensor, the Fourier transform of the correlation $\overline{u_i(t)u_j(t+\tau)}$, whereas F_q is defined as the Fourier transform of $\overline{u_j(t)u_j(t+\tau)}$ (with summation on indices) or $\overline{\tilde{q}(t)\cdot\tilde{q}(t+\tau)}$.

turbulent velocity fluctuations in the same manner as $\overline{\theta^2}$ represents the average strength of the temperature fluctuations. A possible analogy between $\overline{q^2}$ and $\overline{\theta^2}$ can be written as

$$\overline{q^2} = k\overline{\theta^2},$$

where the dimensional proportionality constant k may depend on the mean velocity and temperature fields. One possible choice for k was considered in refs. [6, 7], namely

$$\overline{q^2} = B^2\left(\frac{\partial \bar{U}/\partial y}{\partial \bar{T}/\partial y}\right)^2 \overline{\theta^2},$$

where the dimensionless parameter B was found to be remarkably independent of y in the boundary layer [6, 7]. A spectral analogy between $\overline{q^2}$ and $\overline{\theta^2}$ can also be formulated, on the same physical basis, by reference to turbulence structures corresponding to a particular frequency n. On the basis of the previous mathematical and physical arguments, it seems useful to test the validity of the analogy between $F_q(n)$ and $F_\theta(n)$ against experimental data obtained in different turbulent flows. This is the major objective of this paper.

Specifically, the following flows are considered:

(1) Turbulent boundary layer subjected to the sudden application of wall suction.

(2) Turbulent boundary layer with moderately unstable or stable conditions. Of special interest is the atmospheric surface layer for which the Reynolds number is large so that a large separation exists between those parts of the spectra associated with the production and the dissipation of $\overline{q^2}$ (or $\overline{\theta^2}$).

(3) Free turbulent shear flows such as the plane jet and the two-dimensional (2-D) wake. These flows are characterized by markedly coherent structures and differ, in an important way, from wall flows. A strong link is maintained, close to the wall, between velocity and temperature fluctuations. This link is consequently present all along the flow. Such a link is absent in free shear flows.

(4) Quasi-homogeneous shear flow with a uniform mean temperature gradient. Absent in this flow are the effects of the wall or the influence of turbulent/non-turbulent interfaces. An important feature of this flow is, as will be outlined in Section 7, the absence of pressure in the transport equation for $\overline{q^2}$.

Available data have been used to test the analogy in the above cases, the choice of data being dictated by the availability of spectra of temperature and each of the three velocity fluctuations. Two other typical flows (natural convection and boundary layer in supersonic flow) are discussed in spite of the lack of complete velocity field measurements; these flows are included here because of the different, at least in the present context, initiation of either velocity or temperature fields.

2. COORDINATES USED FOR PRESENTING SPECTRA

Spectra presented in this paper are shown in the form $nF(n)$ vs n. For this type of presentation, the areas under the spectra are proportional to the variance. For boundary layer flows, the dimensionless frequency $f \equiv ny/\bar{U}$ is used. The spectrum F_β, corresponding to a fluctuation β, is normalized such that

$$\int_0^\infty F_\beta \, \mathrm{d}n = 1.$$

The fluctuation β stands for either a velocity or a temperature fluctuation, i.e. $\beta \equiv u, v, w, \theta$. The spectrum $F_q(n)$ is related to $F_\beta(n)$ by equation (3).

For the free shear flows, the non-dimensional frequency used is $\omega^* = k_1 l$. In the case of natural convection, the dimensional wavenumber k_1 is used. Strictly, different symbols should be used for the functions $F_\beta(n)$ and $F_\beta(k_1)$ or $F_\beta(k_1 l)$; for convenience, this is not done here.

3. SLIGHTLY HEATED TURBULENT BOUNDARY LAYER WITH AND WITHOUT CHANGES IN SURFACE CONDITIONS

The measurements were made in a turbulent boundary layer on a slightly heated plate ($T_w - T_1 =$ 22 K) [7]. At the measurement station ($x = 3.69$ m, longitudinal distance downstream from the transition to turbulence) the main mean flow parameters were: $U_1 = 12$ m s^{-1}, $\delta = 62$ mm, $\delta_2 = 6$ mm, Reynolds number $Re_\delta \simeq 50\,000$, $U_* = 48$ cm s^{-1}, $\delta_{th} \simeq \delta$, $T_* = 1$ K. Buoyancy effects were negligible and the ratio y/L was very small since $\delta/L \simeq -0.003$. For these conditions, heat can be considered to be a passive contaminant, at least as far as fluctuations are concerned. Spectral results for many values of y have been comprehensively presented in refs. [6, 7]. We only show here two typical comparisons [Figs. 1(a) and (b)] between the temperature spectrum F_θ and the spectrum F_q.

In the buffer layer [Fig. 1(a)] at $y_* = 16$, the ratio $\overline{u^2}/\overline{q^2}$ is about 0.82 and F_q depends primarily on the longitudinal spectrum F_u. For this reason, the spectra F_θ and F_u are not very different, but it is still evident that F_θ is much closer to F_q than to F_u.

At $y_* = 224$, F_θ and F_u differ markedly because the variance of u has decreased with respect to that of w or v. However, F_θ is generally close to F_q. F_θ lies slightly above F_q for $f \gtrsim 0.4$. The normalization of the spectra requires that this difference is reflected elsewhere in the spectrum; indeed F_q lies slightly above F_θ over the range $0.01 \lesssim f \lesssim 0.4$ but it should be noted that, in this range, this difference is relatively small compared with the magnitude of F_q or F_θ.

When a turbulent boundary layer is subjected to a sudden change in surface temperature [7] or surface heat flux [11] the relaxation of the internal thermal surface layer is relatively slow. Antonia *et al.* [11]

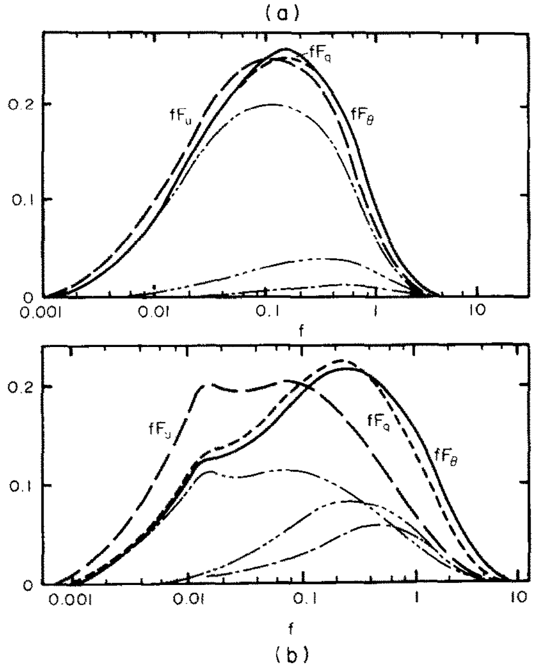

FIG. 1. Temperature and velocity spectra in a slightly heated boundary layer [7]: ——, fF_θ; - - -, fF_q; — —, fF_u; — - — -, $(\overline{u^2}/\overline{q^2})fF_u$; — - - —, $(\overline{v^2}/\overline{q^2})fF_v$; — - - - —, $(\overline{w^2}/\overline{q^2})fF_w$. (a) $y_* = 16$, $\bar{U}/U_1 = 0.438$; (b) $y_* = 224$, $\bar{U}/U_1 = 0.712$.

estimated that a relaxation distance of about $1000\delta_2^1$, where δ_2^1 is the momentum thickness at the location of the change, was required to achieve a self-preserving thermal layer. In the experiment of ref. [7] at a distance of $130\delta_2^1$, the thermal layer thickness δ_{th} was about 0.52δ. At this location, temperature spectra at two values of y/δ ($= 0.065$ and 0.323) compared very well [6] with corresponding spectra obtained in the self-preserving thermal layer at identical values of y/δ (or y/δ_{th}) when the momentum and thermal layers had nearly coincident origins. This comparison would suggest that the temperature spectrum is unaffected by the relative origins of the momentum and thermal layers and therefore unaffected by the magnitude of the local temperature gradients. It also underlines that in a spectral sense θ depends primarily on $\tilde{q}$. Consequently, the analogy between F_q and F_θ is also unaffected by the change in surface condition since F_q is unchanged, at a given value of y/δ. An important corollary is that the spectral analogy does not seem to depend on the mean gradients, in contrast to the Reynolds analogy.

The effect of wall suction on the non-isothermal, self-preserving turbulent boundary layer was studied by Verollet [12], Fulachier and Dumas [6], Verollet *et al.* [13] and Fulachier *et al.* [14]. Suction was applied at a distance $x' = 3.05$ m downstream from the transition to turbulence. The suction ratio, $-\rho_w V_w/\rho_1 U_1 = 0.0030$, is large enough to strongly increase the momentum and heat transfers at the wall, and decrease the production rates of fluctuation variances. The friction coefficient and the Stanton number are about twice as large as for no suction. At the measurement station ($x = 3.69$ m), the flow characteristics with suction were $U_1 = 12$ m s^{-1}, $\delta \simeq 57$ mm, $U_* = 67$ cm s^{-1}, $\delta_{th} \simeq \delta$, $T_* = 1.3$ K.

In Figs. 2(a)–(c) spectra at three typical distances from the wall are shown. Close to the wall [Fig. 2(a)] F_θ and F_u are rather similar; measurements of v and w have not been made in this region. The spectra in Fig. 2(b) have been obtained at the same distance as in the boundary layer with no suction [Fig. 1(b)]: it is evident that F_θ is significantly closer to F_q than to F_u. Also, in contrast to Fig. 1(b), fF_u does not exhibit a plateau, i.e. the f^{-1} power-law variation of F_u, as given by Tchen [15], is not observed. There is also no f^{-1} variation in either F_q or F_θ. In the outer part of the boundary layer [Fig. 2(c)], contributions from each velocity component to the total variance are nearly equal ($\overline{u^2}/\overline{q^2} = 0.39$, $\overline{v^2}/\overline{q^2} = 0.30$, $\overline{w^2}/\overline{q^2} = 0.31$), but the longitudinal component, u, contributes mainly to low frequencies while the contribution of v or w is more

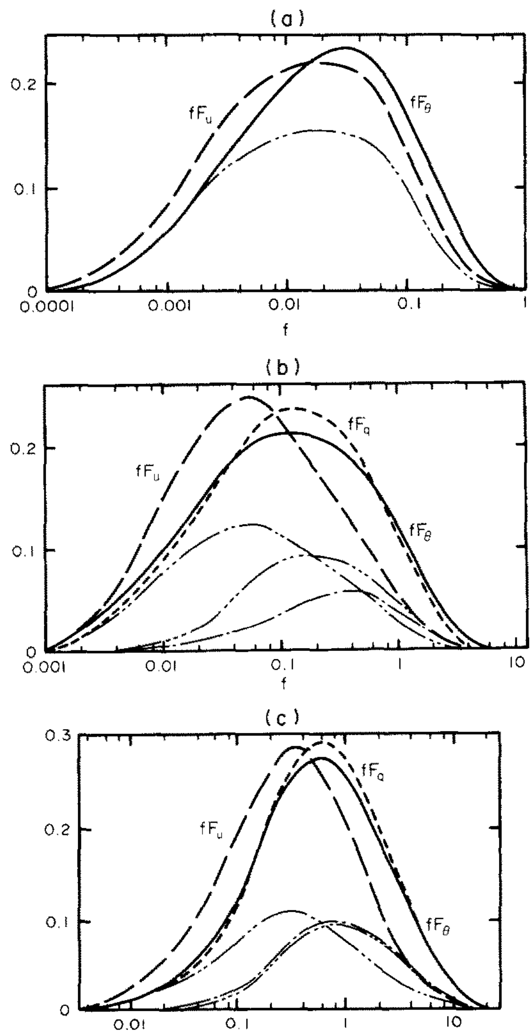

FIG. 2. Temperature and velocity spectra in a slightly heated boundary layer with wall suction [7]. Symbols as per Fig. 1. $\rho_w V_w/\rho_1 U_1 = -0.0030$. (a) $y/\delta = 0.016$, $\bar{U}/U_1 = 0.663$; (b) $y/\delta = 0.113$, $\bar{U}/U_1 = 0.783$; (c) $y/\delta = 0.726$, $\bar{U}/U_1 = 0.987$.

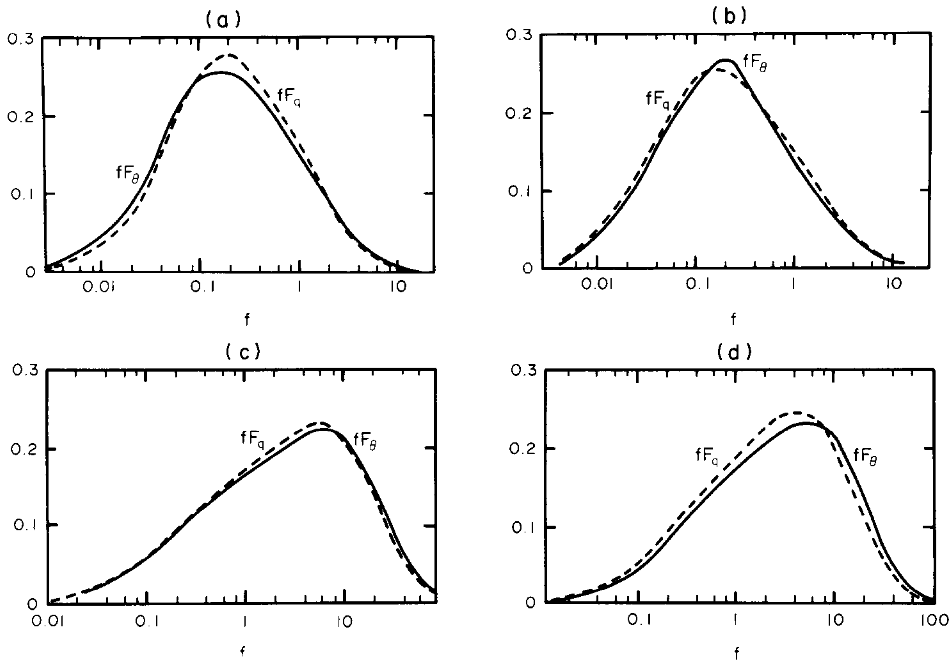

FIG. 3. Temperature and velocity spectra in a moderately unstable boundary layer over a rough wall [21]: ——, fF_θ; ---, fF_q. Smooth: (a) $y/L = -0.19$, $y/\delta = 0.4$, $Re_\delta = 52000$; (b) $y/L = -0.50$, $y/\delta = 0.4$, $Re_\delta = 33000$. Rough: (c) $y/L = -0.07$, $y/\delta = 0.5$, $Re_\delta = 84000$; (d) $y/L = -0.41$, $y/\delta = 0.25$, $Re_\delta = 56000$.

dominant at higher frequencies. The spectral analogy is valid (see also refs. [6, 16]).

4. UNSTABLY STRATIFIED TURBULENT BOUNDARY LAYER WITH AND WITHOUT SURFACE ROUGHNESS

The spectral analogy can be tested in a boundary layer with unstable stratification; in contrast to the previous situations, heat can no longer be considered as a passive contaminant. Schon's [17] experiments in a thermal boundary layer with unstable stratification corroborated approximately the spectral analogy. Mestayer and co-workers [18, 19] validated the analogy for a weakly unstable ($y/L = -0.007$) boundary layer flow in the air–sea interaction simulation tunnel described by Coantic *et al.* [20]. Rey [21] investigated the same boundary layer which was used by Schon [17] with the added effect of wall roughness.

F_θ and F_q are compared in Fig. 3 in four typical cases: smooth wall with slightly unstable (case a) or strongly unstable (case b) stratification or rough wall with almost neutral (case c) or strongly unstable (case d) conditions. In all four cases, the spectral analogy applies but in Rey's experiments, F_u was not very different from F_θ.

5. STABLY STRATIFIED ATMOSPHERIC SURFACE LAYER

For the laboratory boundary layers discussed so far, the turbulent Reynolds number Re_λ is relatively small. The maximum value of $Re_\lambda \equiv \overline{u^2}^{1/2}\lambda_u/\nu$, where λ_u is the longitudinal microscale) was about 600 in the case of Mestayer [18]. It was therefore of interest to consider the atmospheric surface layer where Re_λ is often one order of magnitude larger than in the laboratory.

To this purpose, we have used the well-known spectral data for u, v, w, and θ presented [22], within the framework of similarity theory, by Kaimal *et al.* [23]. Different values of y/L, in the range 0_+† $\leqslant y/L \leqslant +2$, are considered. These curves were deduced from wind and temperature fluctuation data obtained in the 1968 AFCRL (Air Force Cambridge Research Laboratories) Kansas experiments carried out over a flat, uniform site described in ref. [24]. Spectra for unstable stratifications ($y/L < 0$) have not been considered as they do not show a systematic dependence on y/L, but tend to cluster in a random fashion within a significant region‡ of the log–log plots given in ref. [23].

Figure 4 shows the comparison between the temperature spectrum F_θ, the longitudinal velocity spectrum F_u and F_q for values of y/L ranging from 0_+ to 0.5. In particular, the agreement between F_q and F_θ is good for $y/L = 0.1$, 0.3 and 0.5 [Figs. 4(b)–(d)]. The agreement for $y/L = 0_+$ is not as good as in the more stable cases. It is likely, however, that in the approach towards neutral conditions, the temperature signal becomes smaller rendering an accurate determination

† The subscript + denotes that neutral conditions are approached from the positively stable side.

‡ The spectral curves converge to a single universal curve in the inertial subrange for $f \simeq 0.1$ in the case of u and w and for $f \simeq 1.0$ in the case of v.

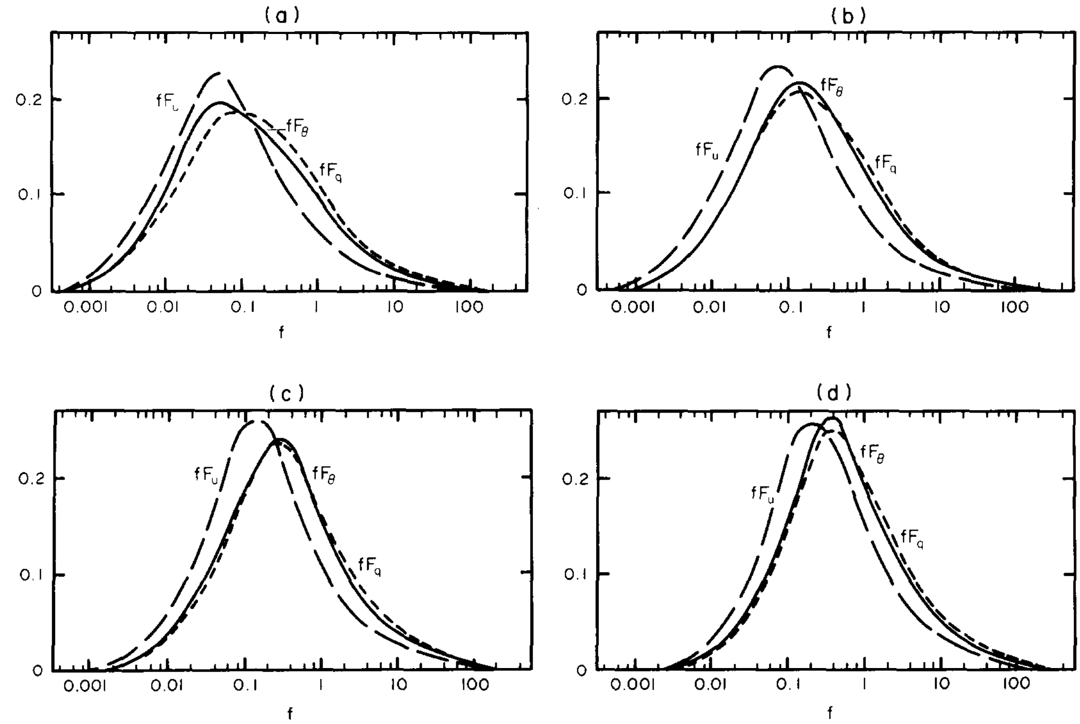

FIG. 4. Temperature and velocity spectra in a slightly stable atmospheric surface layer [23]: ——, fF_θ; ---, fF_q; — —, fF_u. (a) $y/L = 0_+$; (b) $y/L = 0.1$; (c) $y/L = 0.3$; (d) $y/L = 0.5$.

of the statistics of θ difficult. F_θ differs markedly from F_u but lies close to F_q.

When the spectra of Fig. 4 are re-plotted in log–log coordinates (the representation used by Kaimal *et al.* [23]) the inertial subrange extends to a value of f as large as 50. For higher frequencies, the spectral contribution to the variances is small (Fig. 4). As a result of the large Reynolds number of this experiment, the contribution to the total variance from the dissipative subrange is expected to be negligible. This would tend to explain the relatively good agreement between F_θ and F_q for the largest values of f.

When the stability is very strong ($y/L > 1$) the nature of the flow changes. In particular, the possible existence of internal waves which do not contribute to heat transport should be taken into account [25, 26]. Figure 5 clearly shows that the difference between F_q and F_θ becomes more pronounced as y/L increases from 1 to 2.

6. PLANE JET AND WAKE

We consider here two free shear flows which, as noted earlier, differ from the boundary layer in an important way. For the boundary layer, the presence of a heated wall ensures the continuous introduction, at any value of x, of the passive contaminant and acts as an eraser of flow memory. In the free shear flows, neither effect is present.

Spectra of u, v, w were measured [27] in the self-

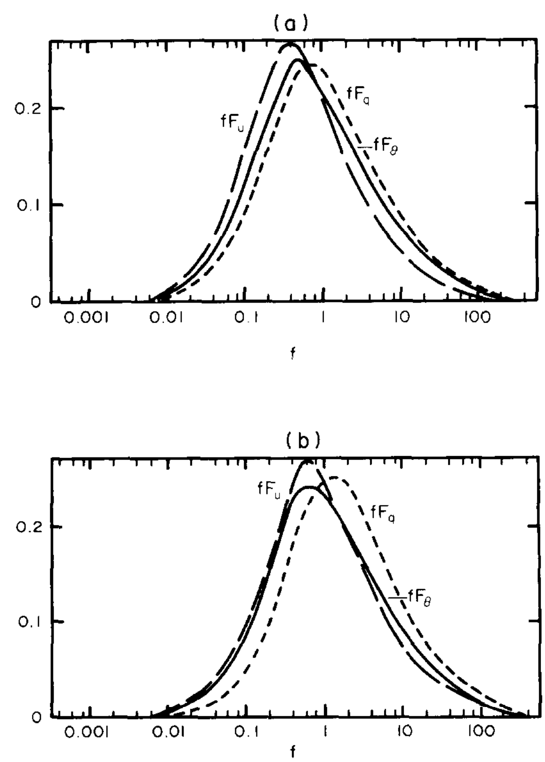

FIG. 5. Temperature and velocity spectra in a strongly stable atmospheric surface layer [23]: ——, fF_θ; ---, fF_q; — —, fF_u. (a) $y/L = 1$; (b) $y/L = 2$.

preserving region ($x/d \gtrsim 20, d = 12.7$ mm) of a plane jet at a Reynolds number $Re_d \simeq 8000$. Spectra of u, v, w at $x/d = 40$ and $y/L_u = 0.5$ are shown in Fig. 6 weighted by the contributions of the individual fluctuations to the variance $\overline{q^2}$. The circular frequency ω is here normalized by the self-preserving scales L_u and U_0, the mean velocity on the centreline. Spectra for v, u, and θ exhibit a peak at $\omega^* = \omega L_u/U_0 \simeq 0.7$.

Space–time correlations of u, v, θ measured in ref. [27] supported the alternate appearance on opposite sides of the centreline of counter rotating structures. The contribution of these structures to the average momentum and heat transfer is not yet known. It is nevertheless clear from Fig. 6 that the comparison between F_q and F_θ is more appropriate than a comparison between F_θ and F_u. The comparison between F_q and F_θ has been tested and found to be adequate at other values of y/L_u in the self-preserving region [28]. It is also satisfactory in the interaction region of the jet. The results of Antonia *et al.* [28] show that on the axis the five spectra F_u, F_v, F_w, F_q, and F_θ are nearly similar. In particular, contributions to $\overline{u^2}$ and $\overline{v^2}$ from F_u and F_v are maximum in the same frequency range. Further, on the axis, the contributions of $\overline{v^2}$ and $\overline{w^2}$ to $\overline{q^2}$, whilst smaller than $\overline{u^2}$, are more important than those measured away from the axis.

Spectra of u have been measured by Uberoi and Freymuth [29] in the turbulent wakes behind circular cylinders over a relatively wide range of x/d and Re_d. They also reported spectra on the centreline of all three fluctuations at $x/d = 200$ and $Re_d = 2160$. These spectra and the spectrum (also on the centreline) of the temperature, given by Freymuth and Uberoi [30] at $x/d = 1140$ and $Re_d = 960$, were used to compare F_q and F_θ in Fig. 7. F_θ is in much closer agreement with F_q

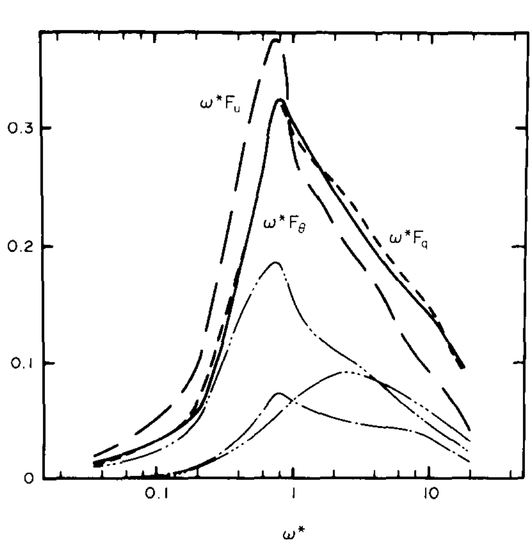

FIG. 6. Temperature and velocity spectra in the plane jet [28]: ——, ω^*F_θ; ---, ω^*F_q; ——, ω^*F_u; ————, $\omega^*F_u(\overline{u^2}/\overline{q^2})$; ———, $\omega^*F_v(\overline{v^2}/\overline{q^2})$; ——–—, $\omega^*F_w(\overline{w^2}/\overline{q^2})$. $x/d = 40$; $y/L_u = 0.5$; $Re_d = 8000$.

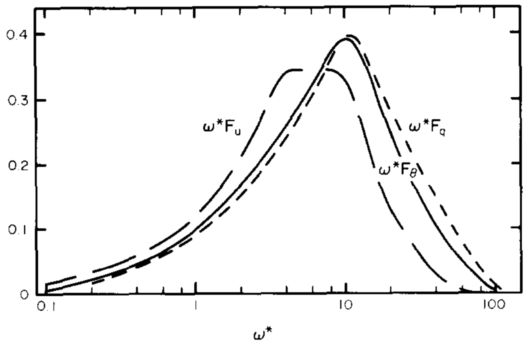

FIG. 7. Temperature and velocity spectra in the wake of a slightly heated cylinder [29, 30]: ——, ω^*F_θ; ---, ω^*F_q; – – —, ω^*F_u. $x/d = 1140$; $y = 0$; $Re_d = 960$.

than with F_u. It should be noted that although the temperature spectrum was obtained at a different position and Reynolds number than the velocity spectra, similarity of the low wavenumber end of the spectrum was established, with respect to either x/d or Re_d, by Uberoi and Freymuth. Further, the F_u spectrum measured by Freymuth and Uberoi at $x/d = 1140$ and $Re_d = 960$ is in agreement with that of Fig. 7 when similarity scales are used for the comparison. It should also be mentioned that F_v shows a spectral peak ($\omega^* = \omega l/U_1 \simeq 10$). It seems likely that, as in the case of the jet, this peak corresponds to the passage frequency of organized structures.

7. NEARLY HOMOGENEOUS TURBULENT SHEAR FLOW

Tavoularis and Corrsin [31, 32] superposed a reasonably uniform mean temperature gradient upon a nearly homogeneous turbulent shear flow in a wind tunnel. One of the aims of their investigation was to study the transport of heat as a passive scalar in a turbulent flow with constant $\partial\bar{U}/\partial y$ and $\partial\bar{T}/\partial y$ and nearly homogeneous, in a transverse direction, velocity and temperature fluctuation fields. Temperature and all three velocity fluctuations were measured at several distances x from the heating rods plane. We are concerned here with the one-dimensional (1-D) spectra of u, v, w, and θ. The results at $x/h = 11$, where h is the height or width of the shear flow generator, are shown in Fig. 8. At this location, the contributions of u, v, and w to $\overline{q^2}$ are equal to 54, 18, and 28%, respectively.

The spectrum of temperature appears to be situated vis-à-vis the u and v spectra in much the same way as it is in the boundary layer (at least when the distance from the wall is sufficiently large). This behaviour and spectral coherences measured by Tavoularis and Corrsin [32] tend to suggest that θ is relatively more influenced by v at high frequencies and by u at low frequencies. The spectrum F_q is in good agreement with F_θ over a significant range of frequencies.

When $\tau = 0$, equations (1) and (2) reduce to equations for the temperature variance and the turbulent energy.

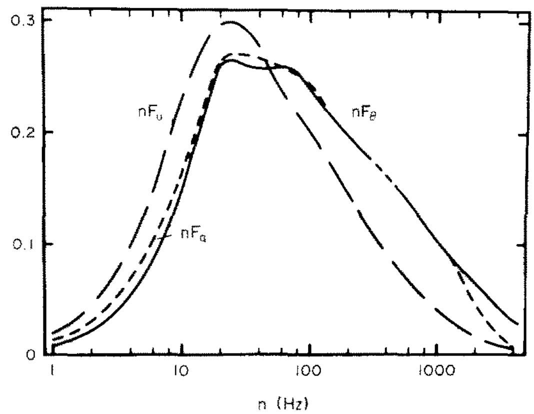

FIG. 8. Temperature and velocity spectra in a nearly homogeneous turbulent flow with uniform mean velocity and mean temperature gradients [31]: ——, nF_θ; ---, nF_q; — —, nF_u. $x/h = 11$.

While, in the present context, these equations are of less interest than equations (1) and (2), it is worth pointing out that in the experimental realization of a quasi-homogeneous shear flow [31, 33], working approximations to the equations were found to be

$$\bar{U}\frac{\mathrm{d}(\overline{\theta^2}/2)}{\mathrm{d}x} \simeq -\overline{\theta v}\frac{\partial \bar{T}}{\partial y} - \varepsilon_\theta, \tag{4}$$

and

$$\bar{U}\frac{\mathrm{d}(\overline{q^2}/2)}{\mathrm{d}x} \simeq -\overline{uv}\frac{\partial \bar{U}}{\partial y} - \varepsilon, \tag{5}$$

where

$$\varepsilon_\theta = \alpha\overline{(\partial\theta/\partial x_i)(\partial\theta/\partial x_i)} \quad \text{and} \quad \varepsilon = \nu\overline{(\partial u_i/\partial x_j)(\partial u_i/\partial x_j)}.$$

The LHS of equations (4) and (5) are non-negligible since both $\overline{\theta^2}$ and $\overline{q^2}$ increase with distance x. The pressure term does not appear in equation (5) although it remains important in the equations for the individual velocity fluctuations. One of the assumptions made in refs. [31, 33] was that velocity and pressure fluctuations were statistically homogeneous in any (y, z) plane. As pointed out by these authors however, transverse homogeneity is strictly impossible in view of the dependence on y of the LHS of equations (4) and (5); in their experiment, transverse homogeneity is only an approximation. The non-appearance of the pressure term in equation (5) and presumably in equation (2), for this particular case, places the spectral analogy on a formal basis.

8. NATURAL CONVECTION AND SUPERSONIC FLOWS

In the flows considered above, heat was introduced, at least initially, independently of the velocity field and vice versa. It is of some relevance to consider two experimental situations where there is dependence, at least initially, between the temperature and velocity fields. In the case of natural convection, the velocity is generated by the thermal field. In the case of a supersonic turbulent flow without external heat transfer, the temperature field is generated by the velocity field.

Doan Kim Son [34] has investigated the natural convection from a vertical heated plate. His spectra of u and θ, obtained in the fully developed turbulent zone for a large value of the Grashof number Gr are shown in Fig. 9, for two values of y. Close to the wall [Fig. 9(a)] there is reasonably close agreement between the two spectra, as is found in the boundary layer [Fig. 1(a)]. Away from the wall [Fig. 9(b)] the departure between the u and θ spectra is significant, as found in the boundary layer. Since the v and w fluctuations were not measured, no conclusion can be drawn about the spectral analogy in this case. If we suppose that the analogy does hold, the similarity between the results of Figs. 9 and 1 would tend to suggest that, as for the case of the low-speed slightly heated boundary layer, the importance of v and w increases with distance from the wall.

For a turbulent boundary layer on an adiabatic wall in supersonic flow, the heat transfer is generated by the velocity field. The link between temperature fluctuations and the longitudinal velocity component is dictated by the total enthalpy relation which, in linearized form, can be written as

$$\theta_0 = \theta + (\gamma - 1)M^2\frac{\bar{T}}{\bar{U}}u.$$

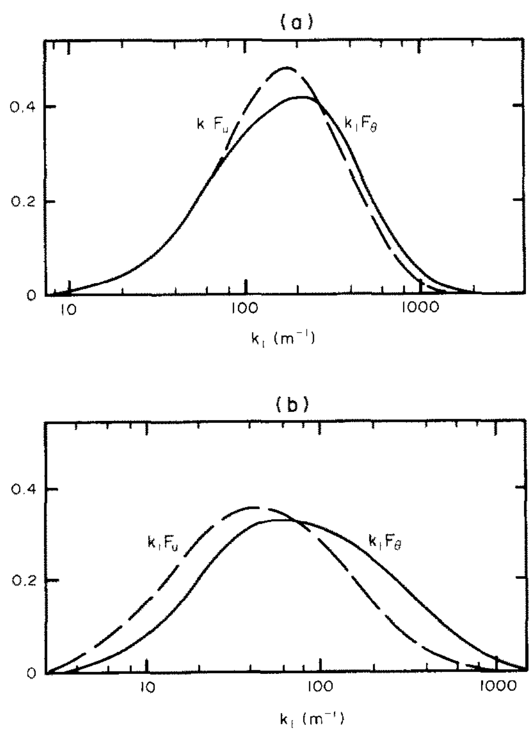

FIG. 9. Spectra of temperature and of the velocity u in natural convection [34]: ——, k_1F_θ; — —, k_1F_u. (a) $y = 0.6$ mm; (b) $y = 60$ mm.

Since θ_0 is small in magnitude and weakly correlated with θ and u, the above relation indicates that θ and u are strongly correlated. In Young's [35] strong Reynolds analogy, it is assumed that $\theta_0 \equiv 0$, which implies that the correlation coefficient between θ and u is -1. Accordingly, the measurements [36–38] indicate that this coefficient is about -0.8, significantly larger in magnitude than the value ($\simeq -0.5$) obtained in the central region of a heated low-speed boundary layer [7]. The magnitude of the filtered correlation coefficient is also in the neighbourhood of 0.8 and exhibits little variation with frequency [38]. However, although spectral measurements in supersonic flow are of a relatively poor accuracy, Bestion's results tend to indicate that the spectra of θ and u differ in similar fashion to the low-speed case (Fig. 10). Further, the ratio of integral time scales, relative to θ and u, is also the same as that in the low-speed boundary layer. Finally, the magnitude of the correlation coefficient between θ and v determined by Bestion [38] in a supersonic flow is of the same order as that obtained in a low-speed thermal boundary layer. The previous information, with due allowance for the experimental uncertainty, lead us to believe that in the supersonic flow θ is, as for the subsonic flow, under the influence of the fluctuation velocity vector $\tilde{q}$, although in the supersonic case u plays a rather special role in the transport of heat. No conclusive statement can be made on the question of the spectral analogy since the four spectra F_θ, F_u, F_v, and F_w are not available; however, the above remarks do not invalidate the spectral analogy.

9. CONCLUSIONS

The analogy between spectral distributions of the temperature variance and the turbulent kinetic energy, previously established in a slightly heated turbulent boundary layer over a smooth wall with zero pressure gradient, has been confirmed in boundary layers with quite different boundary conditions. It has also been confirmed in the high Reynolds number atmospheric surface layer for conditions ranging from near neutral to moderately stable ($0 \gtrsim yL \gtrsim 0.5$). With a further increase in stability, the difference between the temperature spectrum and the spectral distribution of the turbulent energy becomes more pronounced.

In the self-preserving regions of turbulent plane jets and wakes, the spectral analogy is satisfied. The analogy is also closely verified in the experimental situation of a quasi-homogeneous turbulent flow with constant mean velocity and mean temperature gradients. For this configuration, the transport equations for the time correlations (or their Fourier transforms) of temperature and of the fluctuating velocity vector are formally analogous as the pressure term disappears in the equation for $\overline{\tilde{q}(t)\cdot\tilde{q}(t+\tau)}$. The success of the analogy in nearly all the experimental situations examined seems to imply that the influence of the pressure term, at least in the context of the similarity between F_q and F_θ, may not be too important.

The spectral analogy emphasizes the fact that a scalar, such as temperature, depends not only on one of the velocity components but on all three. When the molecular part of the spectrum does not contribute significantly to the total variance, as is the case for the flows considered here, the spectral analogy is satisfied not only for wall-bounded flows but also in free shear flows and quasi-homogeneous flows.

For the classical flows studied in this paper, the departure from unity of the turbulent Prandtl number Pr_t is small in the case of wall-bounded flows but large for the free shear flows. In contrast, the spectral analogy works well in these flows. It seems therefore reasonable, from both mathematical and physical points of view, to seek a relationship, not between momentum and heat fluxes, as is the case with the Reynolds analogy, but preferably between the turbulent kinetic energy and the temperature variance [6, 7]. From a spectral point of view, this is equivalent to seeking a relationship between the temperature spectrum F_θ and a spectrum such as F_q, in preference to one between the co-spectra corresponding to the momentum and heat fluxes [39]. A spectral relationship between F_θ and F_q should be a good candidate for inclusion in spectral closure models.

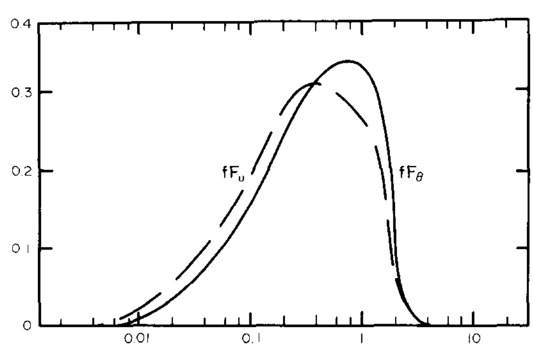

FIG. 10. Temperature and velocity spectra in a supersonic flow turbulent boundary layer [38]: ——, fF_θ; — —, fF_u. $y/\delta = 0.3$; $M = 2.3$.

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ANALOGIE SPECTRALE ENTRE LES FLUCTUATIONS DE TEMPERATURE ET DE VITESSE DANS PLUSIEURS ECOULEMENTS TURBULENTS

Résumé—L'analogie entre les distributions spectrales de variance de température et d'énergie cinétique turbulente, établie par Fulachier et Dumas dans le cas d'une couche limite turbulente avec gradient de pression nul sur une surface faiblement chauffée uniformément, est vérifiée dans différents écoulements turbulents. Ces écoulements incluent une couche limite soumise à une brusque succion pariétale, une couche limite faiblement instable sur une paroi rugueuse, la couche de surface atmosphérique à très faible nombre de Reynolds et avec différentes conditions de stabilité, un jet plan, un sillage plan et une turbulence presque homogène avec une vitesse moyenne uniforme et des gradients de température moyenne. L'analogie s'applique bien sauf dans la couche de surface atmosphérique quand les conditions deviennent plus stables. Dans le cas de la convection naturelle et des écoulements à couche limite supersoniques, l'information sur le champ de vitesse est incomplet mais les résultats n'invalident pas l'analogie.

SPEKTRALE ANALOGIE ZWISCHEN TEMPERATUR- UND GESCHWINDIGKEITSSCHWANKUNGEN IN VERSCHIEDENEN TURBULENTEN STRÖMUNGEN

Zusammenfassung—Die Analogie zwischen den spektralen Verteilungen der Temperaturvarianz und der turbulenten Bewegungsenergie, die von Fulachier und Dumas für den Fall einer turbulenten Grenzschicht ohne Druckgradient an einer gleichmäßig schwach beheizten Fläche eingeführt wurde, wird an verschiedenen turbulenten Strömungen überprüft. Diese Strömungen beinhalten: eine Grenzschicht mit plötzlicher Wandabsaugung, eine mäßig instabile Grenzschicht über einer rauhen Wand, die atmosphärische Oberflächenschicht bei sehr hohen Reynolds-Zahlen mit unterschiedlichen Stabilitätsbedingungen, einen ebenen Strahl, eine ebene Nachlaufströmung und nahezu homogene Turbulenz mit gleichmäßigen Mittelgeschwindigkeits- und Mitteltemperaturgradienten. Die Analogie bewährt sich gut, außer im Fall der atmosphärischen Oberflächenschicht mit zunehmend stabiler werdenden Bedingungen. Im Fall der freien Konvektion und der Überschall-Grenzschichtströmungen ist die Information über das Geschwindigkeitsfeld unvollständig, die verfügbaren Ergebnisse entkräften die Analogie jedoch nicht.

СПЕКТРАЛЬНАЯ АНАЛОГИЯ МЕЖДУ ПУЛЬСАЦИЯМИ ТЕМПЕРАТУРЫ И СКОРОСТИ В КЛАССЕ ТУРБУЛЕНТНЫХ ТЕЧЕНИЙ

Аннотация—Аналогия между спектральными распределениями интенсивности пульсаций температуры и кинетической энергии турбулентности, установленная Фулашье и Дюма для турбулентного пограничного слоя с нулевым градиентом давления на равномерно умеренно нагретой поверхности, проверена для различных турбулентных течений: в пограничном слое при ступенчатом отсосе на стенке, в умеренно неустойчивом пограничном слое на шероховатой стенке, в приповерхностном слое атмосферы при очень больших числах Рейнольдса и при различных условиях устойчивости, в плоской струе и плоском следе, а также при квазиоднородной турбулентности с равномерными средними градиентами скорости и температуры. Аналогия справедлива для всех видов течений за исключением приповерхностного слоя атмосферы при сильно устойчивой стратификации. В случаях естественно-конвективного течения и течения в сверхзвуковом пограничном слое информация о поле скорости оказалась неполной, однако полученные результаты не противоречат концепции аналогии.